Integration of Faith, Learning, and Christian Vocation with First-Year Mathematics Majors

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Our Mission:

The mission of Messiah College is "to educate men and women toward maturity of intellect, character, and Christian faith, in preparation for lives of service, leadership and reconciliation in church and society". Therefore, as faculty in the Mathematical Sciences Department at this college, how we build maturity in our students, not only a mature mathematical intellect, but also maturity of character and Christian faith, reflects our commitment to the mission of the College. Further, our departmental mission statement includes the objective "to challenge students to live out their faith in their vocation as they become servant leaders in society, church, and the world". While all of us in the department agree that our students should be challenged to live out their faith in their vocation, we realize that to include this objective in our mission statement, and to assess our effectiveness in accomplishing the objective, means much more than references to faith in class devotionals or personal interaction with students. We believe that in order for our students to think seriously about their faith and vocation in the context of mathematics, they must discuss these issues, and think and write about them, throughout their college years, as an integral part of the curriculum of their major. Our goal in this paper is to describe a text we are writing that is intended to help first-year mathematics majors learn about Christian integration and vocation as they develop broad, mature mathematical thinking skills. We intend in future years to expand this text to address related issues from a perspective appropriate to the more mature student.

So how does one introduce a first-year mathematics major to the idea of Christian vocation? First, we should note that much has been written about Christian scholarship and vocation in general and also about a Christian perspective on mathematics. The texts *Scholarship & Christian Faith: Enlarging the Conversation* (Jacobsen & Jacobsen, 2004) and *Mathematics in a Postmodern Age: A Christian Perspective* (Howell & Bradley, 2001), are noteworthy examples. However, little has been written at a level that is accessible to the first-year mathematics student. In fact, when we perused the math courses in the 2004 Messiah College catalog, our department found that only in our senior mathematics seminar was there any explicit reference in the course description to moving students toward a maturity of Christian faith. This may be in part because faculty hesitate to include an objective in a course without consistent materials to help guide the class toward accomplishing it.

Second, it is important to recognize that there are many ways to think about faith-integration and Christian vocation. In "Faith-Learning Integration: An Overview" (1992), William Hasker proposes several approaches to the integration process. Further, he argues that the effectiveness of each approach is dependent upon both the discipline and the individual undertaking the process. That is, some approaches to the process are better suited to certain disciplines than others, and the professional interests of an individual may make one approach more desirable to that individual than the others. Whatever the approach, the underlying principle and the ultimate goal of this process is for the participant to think holistically about life and in particular, vocation (or discipline) and faith. In light of this, we believe that any text that introduces students to this process must meet several objectives. It should challenge students to make connections between their studies and their faith, incorporate a number of different perspectives on Christian vocation, and expose students to a variety of examples of faith-integration. In addition, throughout the text, students should be challenged with mathematical content that is classical and rigorous. As students become stronger mathematicians, they have a clearer picture of the discipline they are to view from a Christian perspective. A text that meets these objectives with first-year students has the potential to equip those students with the necessary tools to think about their vocation as it relates to their faith, throughout their education and beyond their undergraduate years.

Our Course:

The text we are writing is not limited to use only in a first-year mathematics seminar like the one we offer at Messiah College. Since this course is where it will first be used, though, we begin with a brief description of MAT 194: First Year Mathematics Seminar, a course that is required for all of our mathematics majors. This two-credit course is offered each spring for twelve weeks, meeting twice a week for one hour, and the majority of mathematics majors take it in their first year. The course description has traditionally been intentionally vague ("Overview of mathematics major; selected topics from finite mathematics and mathematical reasoning"), in order to give flexibility to faculty members and allow them variability in content. With our departmental mission statement in mind, we intend to add some consistency from year to year, continuing as in the past to give an overview of the mathematics major, but focusing more on mathematical reasoning and including a significant component devoted to a Christian perspective on vocation.

Our Text:

The structure and topics of our text, which is very much in draft form as of this writing, can best be described by the following Table of Contents:

Chapter 1: Mathematics and Certainty

Reasoning, Logic, and Truth

1.1: Inductive and Deductive Reasoning
1.2: Symbolic Logic and Truth Tables
1.3: Methods of Proof
1.4: Axiomatic Systems

Perspectives on Certainty and Faith

1.5: A Christian Perspective on Reasoning
1.6: Certainty and the Supernatural

Chapter 2: The Usefulness of Mathematics

Mathematical Work and Careers

2.1: Types of Mathematical Work

Christian Vocation

2.2: Christian Vocation and Mathematics

As the table of contents indicates, significant mathematical knowledge and classical reasoning methods are emphasized in each chapter. Typically, the mathematical content, which includes written exercises for students to complete, leads into the consideration of a Christian perspective on the same theme, often incorporating a variety of readings by both Christian and non-Christian authors. In many cases, the readings are not of a mathematical nature, but they connect religious thought to the mathematical theme of the chapter. Each chapter includes discussion questions and writing assignments in which students are asked to clearly articulate their own developing understanding of issues in mathematics from a Christian perspective.

A more detailed description of Chapter 1 now follows, to give readers a sense of the content and style of the text.

Mathematics and Certainty (Chapter 1)

The goal of this first chapter is to teach students basic concepts in mathematical reasoning, including types of reasoning, truth and equivalent statements of truth, axiomatic systems, and classical methods of proof. First-year students are typically taking a course in the calculus sequence, and they have not necessarily seen even an introduction to broader mathematical thought. We see these topics as an important part of the foundation we want them to have when they reach their upper-division courses. And from a Christian perspective, an understanding of mathematical reasoning and truth leads naturally into a discussion about universal certainty, the role of the supernatural, and the scriptural mandate to worship God with our minds.

Here is what the students read as they begin Chapter 1:

Overview

How do mathematicians know what it is they claim to know? Most first-graders will tell you with assurance that 8+5=13. High school geometry students not only know, but can prove, that the sum of the angles in any triangle is 180 degrees. How do we know that the first-grade student is correct? What does it mean to say that the high school student has proved that a given statement is true? In this chapter we will answer these questions, with a focus on methods of reasoning and proof, beginning with a look at inductive and deductive reasoning.

The mathematical knowledge we have today was not established overnight. The serious mathematics student knows that subjects like calculus and geometry have taken years, in fact centuries, to develop. The great mathematicians of the past used the work of their predecessors, along with the human ability to reason both inductively and deductively, to lay a solid mathematical foundation for today's student. A modern student's quest to rediscover this mathematical knowledge is not unlike that of the great mathematicians who have gone before her. It takes time to develop the ability to prove that the sum of the angles in any triangle is 180 degrees. This level of reasoning ability does not spontaneously appear in the 10^{th} grade. The ability to do this type of proof actually began developing long before the geometry class, most likely in elementary school. And for most people, the foundation for this thinking ability is inductive reasoning.

This overview is followed by an introduction to inductive and deductive reasoning, which leads to the following exercise.

Exercise 1: Inductive Reasoning and Hasty Generalization

When n points are placed on a circle, how many chords are necessary to connect each pair of points by a chord? How many regions are created interior to the circle? For example, if there are two points on the circle as shown above, then the number of chords is 1 and the number of regions is 2.

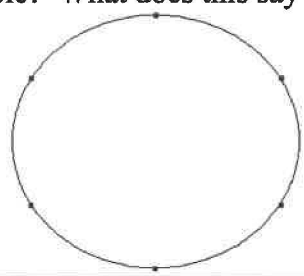
a) Complete the following table, by adding points and chords to the circle above.

Number of points	Number of chords	Number of regions
1	-	1
2	1	2
3		
4		
5		

b) Based on the information in the table, make a conjecture as to the number of chords necessary to connect each pair of points when the circle contains n points.

c) Based on the information in the table, make a conjecture as to the number of regions inside the circle when n points are connected by pairs.

d) Connect each pair of points in the figure below with a chord. How many regions are inside the circle? What does this say about your conjecture in (c)?



Exercise 1 illustrates that inductive reasoning alone is not sufficient to establish a conjecture as true. This exercise is included in the body of the chapter. Other exercises are placed at the end of the chapter, for instructors to assign for independent work. The following is a supplementary exercise on deductive reasoning.

Exercise 5 (Supplementary):

Given the following three facts in Euclidean geometry, construct a deductive argument that the sum of the angles in any triangle is 180°.

Fact 1: If two parallel lines are cut by a transversal, then each pair of corresponding angles is congruent.

Fact 2: Vertical angles (opposite angles formed by intersecting lines) are equal.

Fact 3: A straight angle (the sides of which form a straight line) is an angle of 180°.

In sections 1.2, 1.3, and 1.4, students are guided in the use of truth tables to evaluate compound statements, they are taught to recognize forms of equivalent statements (such as the contrapositive), and they learn standard methods of proof (induction, proof by contradiction, etc.). Exercises similar in difficulty to those described above are included in each section, and solution guidelines are provided for instructors. More than ten mathematical exercises are currently included in Chapter 1. By the end of the chapter, students who complete all of these will have experience with the following:

- Disproving by counterexample a result that seems (inductively) to be true for all integers
- Understanding and explaining a 'Proof Without Words' (Roger Nelsen's series, published by the MAA)
- Illustrating DeMorgan's Laws in truth tables
- Proving that there are infinitely many prime numbers
- Exploring spherical and hyperbolic geometry and making conjectures in these geometries
- Proving a theorem from a finite set of axioms.

In section 1.5, we consider how correct or faulty reasoning can influence a Christian's beliefs and religious practice, beginning with the following discussion:

<u>Reasoning and a Christian perspective</u>

Human beings can have a flawed sense of reality. In this chapter, we have discussed both deductive and inductive reasoning, and here we would like to suggest ways in which these types of reasoning can help us develop a proper view of reality. First, our study of inductive reasoning identifies errors in reasoning that humans sometimes make. Earlier in this chapter, you may have made the false conjecture that when n points are placed on the circumference of a circle, the number of chords necessary to connect each pair of points is 2^{n-1}. This is an example of a fallacy sometimes referred to as a hasty generalization. A hasty generalization is a conclusion that is based upon an inadequate number of cases or cases that have not been fully analyzed. Our study of mathematics teaches us that although inductive reasoning serves as a powerful tool in making generalizations, it can sometimes lead to wrong conclusions.

The fallacy of hasty generalization is one we must guard against in many contexts in life. As a rather trivial example, if it has not rained in twenty-four days, it is a hasty generalization to assume that it will not rain tomorrow. Similarly, if an individual receives belligerent or annoying treatment from several people of a particular race, it is a fallacy to assume that all members of the race are belligerent or annoying. In each of these examples, a small number of instances suggests a particular conclusion, but the conclusion does not necessarily follow.

Christians must guard against hasty generalization as they study the Bible. For instance, throughout the New Testament, Jesus healed many people who were sick, sometimes before they even asked for healing. Does it follow therefore that Jesus heals all sick people? There are many people today who struggle with physical illness and wonder why God does not provide healing, since He has done it for others. However, if the Bible teaches that God heals all people, or that Jesus healed everyone he came into contact with who was sick, why then didn't he heal all of the people by the pool at Bethesda? The gospel writer indicates that a great number of disabled people were present, but only one man was healed. As another biblical example, before Christ's coming to earth, the writer of Ecclesiastes laments repeatedly that "Everything is meaningless", based on his observation that generations come and go and patterns of injustice and wearisome work continue. Peter comments on this fallacy in 2 Peter 3, starting in verse 4: "[Scoffers] will say 'Where is this 'coming' he promised? Ever since our fathers died, everything goes on as it has since the beginning of creation.' But they deliberately forget that long ago by God's word the heavens existed and the earth was formed out of water and by water. . . . The Lord is not slow in keeping his promise, as some understand slowness. He is patient with you, not wanting anyone to perish, but everyone to come to repentance." (verses 4, 5, and 9)

Our study of inductive reasoning challenges us to look for hasty generalizations in our own reasoning. Does it appear in the form of some prejudice? Or in the narrow way in which we view God? The ability to recognize errors in our own thought processes is essential if we are to have a Christian worldview that is based on truth. Mathematical thinking not only helps us to recognize hasty generalization but also other fallacies in reasoning.

What does our study of mathematics teach us about deductive reasoning? Recall that deductive reasoning begins with certain assumptions, and the conclusions we arrive at are only as reliable as the assumptions they are built upon. Our study of deductive reasoning challenges us to identify the assumptions we make about reality. How do the assumptions we make about God and humanity influence the way we live out our faith?

A deductive study of the Bible looks for principles or axioms about God and faith that are broadly supported in scripture. These axioms are the basis for arguments logically constructed from these principles. The doctrine of inspiration, that the Biblical writers were uniquely inspired by God, is one example of a principle that serves as an axiom for many Christian believers. Based on this axiom, Christians trust that the teachings of the Bible are timeless and sufficient as a foundation for a philosophy of life. Those who do not accept the axiom of the inspiration of Scripture may accept some principles from the Bible, as well as principles from other sources, including other religions. Perhaps you have had conversations, maybe heated arguments, with people whose axioms of faith were different from your own.

What are the axioms of your faith? An individual's answer to this question has a tremendous impact on how he reads and interprets Scripture, and it can lead that individual to conclusions that very much affect his way of life. Certainly a study of both deductive and inductive reasoning is helpful in understanding human thought and errors in human reasoning. We argue that a mathematical understanding of reasoning helps us to formulate a Christian perspective on reality.

Section 1.5 concludes with several questions for class discussion and the following writing assignment:

Writing Assignment

Write a 4-6 page argumentative paper that supports a position you have taken regarding a controversial faith issue, identifying the axioms of your faith that are a basis for your argument.

In the section "Certainty and the Supernatural", we consider the comments of Bertrand Russell, in light of the Scripture passage from Isaiah 55: 8,9:

"For my thoughts are not your thoughts, neither are your ways my ways," declares the Lord. "As the heavens are higher than the earth, so are my ways higher than your ways and my thoughts than your thoughts."

> I wanted certainty in the kind of way in which people want religious faith. I thought that certainty is more likely to be found in mathematics than elsewhere. But I discovered that many mathematical demonstrations, which my teachers expected me to accept, were full of fallacies, and that if certainty were indeed discoverable in mathematics, it would be in a new field of mathematics, with more solid foundations than those that had hitherto been thought secure. But as the work proceeded, I was constantly reminded of the fable about the elephant and the tortoise. Having constructed an elephant upon which the mathematical world could rest, I found the elephant tottering, and proceeded to construct a tortoise to keep the elephant from falling. But the tortoise was no more secure than the elephant, and after some twenty years of arduous toil, I came to the conclusion that there was nothing more that I could do in the way of making mathematical knowledge indubitable (in "Portraits from Memory", as cited in *The Mathematical Experience*, by Davis & Hersh, 1981, page 333).

Through an introduction to non-Euclidean geometry, students learn that mathematics can sometimes provide local certainty within a particular framework, but the global picture may be uncertain. Can the same contrasting perspectives be found in our Christian faith? The readings below explore the ideas of certainty and the supernatural from a faith perspective.

> **Reading and Writing Assignment:**
>
> "Life in Part" and "Rumors" from *Rumors of Another World: What on Earth are we Missing?,* by Philip Yancey, 2003.
>
> Write a 3-5 page argumentative paper that describes the appropriate role of the supernatural in understanding reality. How should mathematicians allow for or factor in rumors of another world, as Yancey suggests?

Chapters 2 and 3

The format of chapters 2 and 3 is similar to that of Chapter 1, so we only give a brief overview of the chapters here. Chapter 2 is entitled "The Usefulness of Mathematics", and in this chapter we introduce students to a variety of mathematical interests (pure vs. applied work, recreational mathematics, mathematics education) and how these interests can lead to a range of careers. The students are asked to read from *101 Careers in Mathematics* (Andrew Sterrett, ed., 1996) and write about their own interests and career plans. Later in the chapter, they read from two Christian authors: portions of *The Will of God as a Way of Life* (Sittser, 2000) and *Courage and Calling: Embracing Your God-Given Potential* (Smith, 1999). Their assignment is to write a persuasive letter to a friend who is unsure about pursuing a career in mathematics because he isn't sure how such a career might impact the world for God. We hope after reading chapter 2 and completing the included exercises and assignments, our first-year students will have confidence that their own strengths and interest in mathematical science can be used to the glory of God.

In Chapter 3, "Mathematics and the Infinite", we discuss infinite processes, infinite series, and paradoxes of the infinite. Typically, students in MAT 194 are concurrently taking Calculus II, in which a more rigorous study of infinite series is taking place. In some significant ways, an understanding of the infinite from a mathematical perspective can contribute to an understanding of theology as we learn in our faith about an infinite God and our relationship to Him. Students read the first chapter of *The Trivialization of God* (McCullough, 1995) and write a 3-5 page paper that addresses how a mathematical understanding of the infinite can contribute to the reverence and awe that Job 38-42 exhorts us to offer to God.

Conclusion:

This first edition of our text is not meant to be a comprehensive treatment of the integration of mathematics, vocation, and the Christian faith. Rather, we see these first three chapters as a starting point, suitable for a two-credit course for first-year students. In the future, we will write chapters that address, from a Christian perspective, mathematics and culture, applied fields such as statistics, and additional mathematical work that is a foundation for graduate study. Ultimately we plan to develop a text that is more complete and therefore suitable for use in a variety of courses within our major, starting in our first-year seminar and continuing through our senior capstone course. We hope that our text will become a meaningful tool toward the mission of our Christian college, to graduate students who live out their faith as mathematicians and in their vocation.

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